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EINSTEIN'S SUGGESTION OF A CONNECTION BETWEEN SCIENTIFIC PRACTICE AND ONTOLOGY. A NEW METAPHYSICS

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> Thinking in dichotomies may be the most venerable (and ineluctable) of all human mental habits. In his *Lifes and Doctrines of the Most Famous Philosophers* (circa A.D. 200) Diogenes Laertius wrote: "Protagoras asserted that there were two sides to every question, exactly opposite to each other."[1]

ABSTRACT

I show that an Einstein's reflection of 1954 about the notion of "space" is substantially equivalent to a reflection about the notion of "set". In addition, this Einstein's writing suggests that the notion of space in theoretical physics enjoys a couple of distinctions in its meaning (one distinction explicitly stated by Einstein, another suggested in a somewhat covert way); and, owing to the above parallelism, also the notion of set in mathematics enjoys a couple of two distinctions in its meaning. Therefore, an ontology of four fundamental meanings of two basic concepts in respectively physical practice and mathematical practice results. Actually, already in his 1905 paper on quanta Einstein had introduces two distinctions and moreover he explicitly had referred to one of them through more accurate notion of a "dichotomy". In addition, these same two dichotomies have been independently recognized by several previous analyses of scientific subjects. By crossing together, the two dichotomies one obtains four models of scientific theory. Therefore, a general metaphysical ontology of four models of scientific theories in both physical practice and mathematical practice results.

Keywords: *Einstein; Space; Set; Dichotomy on the infinity; Dichotomy on the theoretical organization; Four models of scientific theory;*

INTRODUCTION

In the following I will introduce an Albert Einstein's writing[2] that in the past passed almost un-analyzed[3]. It was edited in 1954 as a preface to a celebrated book by a historian of

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[1] Gould S.J. (2002), *The Structure of Evolutionary Theory*, Harvard: Harvard U.P., p. 251.

[2] Einstein A. (1954), "Forward", in Jammer M., *Concepts of Space*, Cambridge MA: Harvard U.P., pp. xv-xvi. https://books.google.it/books?hl=it&lr=&id=R83CAgAAQBAJ&oi=fnd&pg=PP1&dq=einstein+forward+to+Jammer+space+&ots=PSYfcKM46D&sig=ageDXpIjViLLNt3HK6N8ZjoemQg&redir_esc=y#v=onepage&q&f=false.

[3] I only know the comment by Robert Rynasiewicz (1996), "Absolute versus relational space-time: An outmoded debate?", *Journal of Philosophy*, 93, no. 6, pp. 279-306. Through hasty historical appraisals the

physics, David Jammer. There, Einstein tries to explore in an autonomous way a difficult epistemological question about exactly the physical notion which half a century before he drastically changed through his theory of special relativity into the notion of space-time. This event was the starting point of a theoretical revolution in theoretical physics. The 1954 writing offers Einstein's long-time reflection on the notion of space.

In the following sect.s 2-4 an accurate reading of this writing leads to discover that it allows two interpretations, since the word "space" can be also replaced with the notion of "set". This connection shows that within scientists' minds a same ontology[4] may exist through the basic notions of distinct scientific disciplines, although these notions are very different[5]. In sect. 5 the content of the writing is taken into account: Einstein's reflection suggests four philosophical meanings of the notion of space. Sect. 6 recalls that actually, already in his 1905 paper on quanta Einstein had introduced two distinctions and moreover he explicitly had referred to the notion of a "dichotomy" which makes more accurate previous notion of distinction. Remarkably, these same two distinctions-dichotomies have been independently recognized by several previous analyses of scientific subjects[6]. In sect. 7, by crossing together the two dichotomies one obtains four models of scientific theory. Therefore, a general ontology of four models of scientific theories in both physical practice and mathematical practice results.

1. EINSTEIN ABOUT THE FOUNDATIONAL MEANINGS OF THE NOTION OF SPACE. A SUMMARY OF HIS WRITING

I will show that Einstein's analysis of the notion of space substantially applies also to the notion of set. My method is to replace in Einstein's writing the word "space" with the word "set". A consistent reading also representing some relevant aspects of past histories of the two disciplines results.

For a reader's better understanding I anticipate the contents of following Einstein's writing. In a first time Einstein's reflection underlines the importance of the scientific practice in answering certain purely epistemological questions such as the definitions of the basic notions of a theory. In the periods 1 and 2 of the following text, he stresses the

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author claims that "the question […] is no longer a meaningful one". Rather, he seems worry to cancel any reference of this basic notion of theoretical physics to an ontology.

[4] Here the word "ontology" is intended as "the study of the most general features of what there is". Thomas Hofweber (2017), "Logic and ontology", in E.N. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/logic-ontology/, O3. Another definition is "the study of what categiories of entities there are and how they are related to one another". Lowe E.J. (2002), *A Survey of Metaphysics*, Oxford: Oxford U.P., p. 14.

[5] I know only one more connection between two different disciplines, the history of mathematics and the history of Western philosophy after Kant: Marx K. (1983), "On the History of Differential Calculus" in *Mathematical Manuscripts of Karl Marx* (orig. 1888), London: New Park Publications. Available at
1. http://www.marxists.org/archive/marx/works/1881/mathematical-manuscripts. Notice that the common interpretations of this paper charge Marx to have ignored Cauchy's "rigorous reform", whereas he rather anticipated the intuitionist and constructivist re-founding of mathematics (Drago A. (1982), "Le implicazioni teoriche dei manoscritti matematici di Marx", *Testi e Contesti*, n. 8, pp. 107-116).

[6] Drago A. (1988), "A Characterization of Newtonian Paradigm", in Scheurer P.B., Debrock O. (eds.), *Newton's Scientific and Philosophical Legacy*, Kluwer Acad. P.,. 239-252. Drago A. (2012), "Pluralism in Logic. The Square of opposition, Leibniz's principle and Markov's principle, in *Around and Beyond the Square of Opposition*, edited by J.-Y. Béziau and D. Jacquette, Basel: Birckhaueser, pp. 175-189; Drago A. (2017), *Dalla storia della Fisica alla scoperta dei fondamenti della Scienza*. Aracne, Roma 2017 (English translation: *From the History of Physics to the Discovery of the Foundations of Physics*, 2021, https://antoninodrago.com/#pub3).

foundational importance of a reflection on the basic scientific notions; in Einstein's case, the notion of space[7]. He remarks that this kind of reflection is commonly delegated to specialist scholars, but he states that the different meanings of the basic notions may greatly influence scientists' practice. In periods 3 and 4 he distinguishes between the meanings of "(a)" "positional" (i.e. relational or Leibnizian) space and "(b)" "containing" space and even "absolute container" (e.g. the foundational notion of Newton's mechanics). Then, he also distinguishes, although in a covert way, between the two meanings of space as a constructive notion and an idealistic, a priori notion. In a second time, Einstein's reflection suggests disputable historical considerations on the historical development of mechanics. In periods 5-7 he considers the theoretical leap from the notion of space equipped with coordinates (Descartes) to the principle of inertia. Since from here on he suggests disputable historical appraisal, the reader is advised to contrast each his appraisal with an alternative appraisal which will be indicated in the footnotes added to the following text. In a third time he introduces the notion of field as the culmination of his historical view on the notion of space. In per. 8 this notion of field is intended at least in a first time by him as a process of step-by-step interaction and later maybe as an idealistic notion. In per. 9 a final appraisal concerns the insuperability of the notion of field in theoretical physics.

2. EINSTEIN ABOUT THE FOUNDATIONAL MEANINGS OF THE NOTION OF SPACE. AN INTERPRETATION OF HIS TEXT THROUGH THE NOTION OF SET

In the following I present Einstein's text after some preliminary notes. Some periods concern the personal work of the author of the book presented by Einstein, David Jammer. Since they are not interesting our investigation, each of these periods will be replaced by the symbol {**J**}. I number the remaining periods; their numbers are written between square parentheses [ ] at the starting of each period. I add some words between square parentheses [ ] in order to round the meaning of the original text. Moreover, I present the text as it is modified by my substitutions, written in Italic; however, I add between round parentheses {} and written in Italic the corresponding original words. The uncertain correspondences are marked by an asterisk *.

[1] *In order to appreciate* fully the importance of investigations such as the present work of Dr. Jammer one should consider the following points. The eyes of the *mathematician* {[natural] *scientist*} are directed upon those *operative*[8] *mathematical objects* {*phenomena*} which are accessible to *reflection* {*observation*}, upon their apperception and conceptual formulation. In the attempt to achieve a conceptual formulation of the confusingly immense body of *mathematical objects* {*observational data*}, the *mathematician* {[natural] *scientist*} makes use of a whole arsenal of concepts which he imbibed practically with his mother's milk; and seldom if ever is he aware of the eternally problematic character of his concepts. He uses this conceptual material, or, speaking more exactly, these conceptual tools of thought, as something obviously, immutably given; something having an objective value of truth which is hardly ever, and in any case not seriously, to be doubted. How could he do otherwise? How would *a difficult calculation** {*the ascent of a mountain*} be possible, if the use of *the basic mathematical tools* {*hands, legs*} had to be sanctioned step by step on the

[7] Unfortunately, he ignored the analogous analysis by N.I. Lobachevsky in the "Preface" of the book *New Principles of Geometry with Complete Theory of Parallels*, G. B. Halsted (tr.), 1897.

[8] I qualify the mathematical object as an operative one because Einstein refers to non-idealized mathematical objects which instead abound in present mathematical theories.

basis of the *elementary mathematics* {*science of mechanics*}? And yet in the interests of science it is necessary over and over again to engage in the critique of these fundamental concepts, in order that we may not unconsciously be ruled by them. This becomes evident especially in those situations involving development of ideas in which the consistent use of the traditional fundamental concepts leads us to paradoxes difficult to resolve. Aside from the doubt arising as to the justification for the use of the concepts, that is to say, even in cases where this doubt is not in the foreground of our interest, there is a purely historical interest in the origins or the roots of the fundamental concepts. Such investigations, although purely in the field of history of thought, are nevertheless in principle not independent of attempts at a logical and psychological analysis of the basic concepts. But the limitations to the abilities and working capacity of the individual are such that we but rarely find a person who has the philological and historical training required for critical interpretation and comparison of the source material, which is spread over centuries, and who at the same time can evaluate the significance of the concepts under discussion for science as a whole.

{**J**}

[2] In the main he [Jammer] has limited himself — wisely, it seems to me — to the historical investigation of the concept of *set* {*space*}. If two different authors use the words "red," "hard," or "disappointed," no one doubts that they mean approximately the same thing, because these words are connected with elementary experiences in a manner which is difficult to misinterpret. But in the case of words such as "*collection* {*place*}" or "*set* {*space*}", whose relation with psychological experience is less direct, there exists a far-reaching uncertainty of interpretation. The historian attempts to overcome such uncertainty by comparison of the texts, and by taking into account the picture, constructed from literature, of the cultural stock of the epoch in question. The *mathematician* {[natural] *scientist*} of the present, however, is not primarily trained or oriented as a historian; he is not capable of forming or willing to form his views on the origin of the fundamental concepts in this manner. He is more inclined to allow his views on the manner in which the relevant concepts might have been formed, to arise intuitively from his rudimentary knowledge of the achievements of *mathematics* {[natural] *science*} in the different epochs of history. He will, however, be grateful to the historian if the latter can convincingly correct such views of purely intuitive origin.

[3] Now as to the concept of *set* {*space*}, it seems that this was preceded by the psychologically simpler concept of *collection* {*place*}. *Collection* {*Place*} is first of all [*c*)] *a process of immediate gathering* {*a (small) portion of the earth's surface*} identified by a name. The thing whose "*collection* {*place*}" is being specified is a "*operative mathematical object* {*material object or body*}". Simple analysis shows "*collection* {*place*}" also to be a group of *mathematical operative objects* {*material objects*}[9]. Does the word "*collection* {*place*}" has a meaning independent of this one, or can one assign such a meaning to it? If one has to give a negative answer to this question, then one is led to the view that *collection* {*place*} is a sort of order of *operative mathematical objects* {*material objects*} and nothing else. If the concept of *set* {*space*} is formed and limited in this fashion, then to speak of empty *set* {*space*} has no meaning. And because the formation of concepts has always been ruled by instinctive striving for economy, one is led quite naturally to reject the concept of empty *set* {*space*}.

[9] Einstein reports the celebrated polemics between Leibniz and Clark (in the behalf of Newton) about the two meanings of space, respectively relational or absolute.

[4] It is also possible, however, to think in a different way. Into a certain box we can place a definite number of grains of rice or of cherries, etc. It is here a question of a property of the material object "box," which property must be considered "real" in the same sense as the box itself. One can call this property the "*box as a set* {*space of the box*}". There may be other boxes which in this sense have an equally large "*set* {*space*}". This concept "*set* {*space*}" thus achieves a meaning which is freed from any connection with a particular *operative mathematical object* {*material object*}. In this way by a natural [!] extension of "*box as a set* {*box space*}" one can arrive at the concept of an independent (absolute) *set* {*space*}, unlimited in extent[10], in which all *operative mathematical objects* {*material objects*} are contained. Then *an operative mathematical object* {*a material object*} not situated in *set* {*space*} is simply inconceivable; on the other hand, in the framework of this concept formation it is quite conceivable that an empty *set* {*space*} may exist. These two concepts of *set* {*space*} may be contrasted as follows: (a) *set* {*space*} as *membership* {*positional*} quality of the world of *operative mathematical objects* {*material objects*}; (b) *set* {*space*} as container of all *operative mathematical objects* {*material objects*}. In case (a), *set* {*space*} without *an operative mathematical object* {*a material object*} is inconceivable. In case (b), *an operative mathematical object* { *a material object*} can only be conceived as existing in *set* {*space*}; *set* {*space*} then appears as a reality which in a certain sense is superior to the *operative mathematical world* {*material world* [of objects]}. Both *set* {*space*} concepts are free creations of the human imagination, means devised for easier comprehension of our *reflection experience* {*sense experience*}.

[5] These schematic considerations concern the nature of *set* {*space*} *from a descriptive* {*from geometric and from the kinematic*[11]} point of view {, *respectively*}. They are in a sense reconciled[12] with each other by *Cantor's* {*Descartes'*} introduction of the *coordinate of the hierarchy of set's powers* {*coordinate system*}, although this already presupposes the logically more daring *set* {*space*} concept (b). The concept of set {*space*} was enriched and complicated by *Zermelo* {*Newton*}, in that set {*space*} must be introduced

[10] Here Einstein surreptitiously adds one more couple of meanings of the notion of space. He refers to its capability to include an infinite number of material objects. Of course, the infinity may be either potential infinity or actual infinity. Hence, we may attribute to him a further distinction of the notion of space, fugitively mentioned in his words "unlimited in extent". Einstein does not devote further attention to this distinction. As all the physicists of his time Einstein gave a little importance to the contemporary polemics between Hilbert and Brouwer about the indispensability of actual infinity in mathematics or not. It is famous his depreciating appraisal: "What is this frog and mouse battle among the mathematicians? [i.e. Brouwer vs. Hilbert]". In H. Eves (1972), *Mathematical Circles Squared*, Boston: Prindle, Weber and Schmidt, no. 185. A critical analysis of Einstein's position on this subject is presented by van Dalen D. (2001), "The War of the Frogs and the Mice, or the Crisis of the *Mathematische Annalen*, in Wilson R. et al. (eds.), *Mathematical Conversations*, Berlin: Springer-Verlag. However, Einstein was well-aware of the importance o give to physical notions an operative character; see e.g. his critical analysis of the notion of simultaneity in introducing the theory of special relativity.

[11] Actually, Einstein has not previously introduced the notion of time, which would have justified his mentioning the kinematics.

[12] This word is denied by the subsequent word "although" and the content of the following proposition. Although the beginning of the entire reflection are uncertain words "in a sense", here Einstein supports (without arguments) the necessity of the past conception of absolute (Newton's) space, qualified by him as "logically more daring", maybe because he evaluates the relational notion of space as insufficient to achieve the notion of inertia (instead Leibniz, supporting the relational meaning of space, enounced this principle by the simple proposition "indifference of rest and motion"). Actually, the birth of inertia principle required much more preliminary notions. See for instance, Shea W.R. (1983), "Introduction", to Shea W.R. (ed.), *Nature Mathematised*, Boston: Reidel.

as the independent cause of [rather: a merely basic notion for describing] the *Zermelo's choice axiom* {*inertia behaviour of bodies*} if one wishes to give the *Zermelo's axiom* {*classical principle of inertia*} (and therewith the *Zermelo-Fraenckel's axiomatic set theory* {*classical law of motion)*}) an exact meaning. To have realized this fully and clearly is in my opinion one of *Zermelo's* {*Newton's* } greatest achievements. In contrast with *Gauss Kronecker and Brouwer* {*Leibniz and Huygens*}, it was clear to *Zermelo* {*Newton*} that the *set* {*space*} concept (a) was not sufficient to serve as the foundation for the *Zermelo's choice axiom and its axiomatic* {*inertia principle and the law of motion*}. He came to this decision even though he actively shared the uneasiness which was the cause of the opposition of the other two: *set* {*space*} is not only introduced as an independent thing apart from *operative mathematical objects* {*material objects*}, but also is assigned an absolute role in the whole *deductive* {*causal*} structure of the theory[13]. This role is absolute in the sense that *set* {*space*} (as *Zermelo's axiom* {*an inertial system*}) acts[14] on all *operative mathematical objects* {*material objects*}, while these do not in turn exert any reaction on *set* {*space*}.

[6] The fruitfulness of *Zermelo's* {*Newton's*} system silenced these scruples for several decades {*centuries*}. *Set* {*space*} of type (b) was generally accepted by *mathematicians* {[natural] *scientists*} in the precise form of the *Zermelo's axiom* {*inertial system*} encompassing the *continuum hypothesis* {*time*} as well. Today one would say about that memorable discussion: *Zermelo's* {*Newton's*} decision was, in the contemporary state of science, the only possible one, and particularly the only fruitful one[15]. But the subsequent development of the problems, proceeding in a roundabout way which no one then could possibly foresee, has shown that the resistance of *Gauss, Kronecker and Brouwer* {*Leibniz and Huygens*}, intuitively well founded but supported by inadequate arguments, was actually justified[16].

[7] It required a severe struggle to arrive at the concept of independent and absolute *set* {*space*}, indispensable for the development of theory. It has required no less strenuous exertions subsequently to overcome this concept — a process which is probably by no means as yet completed.

{**J**}

[8] The victory over the concept of absolute *set* {*space*} or over that of the *Zermelo's axiom* {*inertial system*} became possible only because the concept of the *operative mathematical object* {*material object*} was gradually replaced as the fundamental concept of *mathematics* {*physics*} by that of the *recursion process** {*field*[17]}. Under the influence of the ideas of *Skolem and Goedel* {*Faraday and Maxwell*} the notion developed that the whole of *mathematical* {*physical*} reality could perhaps be represented as a *recursion process** {*field*} whose components depend on *elementary arithmetical operations** {*four space-time parameters*}. If the *definitions* {*laws*} of this *computable process* {*field*} are in general *equivalent according to Turing-Church thesis* {*covariant*}, that is, are not dependent on a particular choice of the notion of computability {*coordinate system*}, then the introduction of an independent (absolute) *set* {*space*} is no longer necessary. That which constitutes the *set-theoretical* {*spatial*} character of reality is then simply the *computability of the computable*

[13] The following proposition comes back to the descriptive illustration of the notion of absolute space.
[14] Obscure verb.
[15] See footnote no. 12 for Einstein's support to the absolute space in classical theoretical physics.
[16] The justification of this proposition is given in the following two periods.
[17] Conceived, as Faraday did, as a step-by-step interaction.

*process** {*four-dimensionality of the field*}. There is then no "empty" set {*space*} that is, there is no set {*space*} without a computable process* {*field*}.

{**J**}

[9] Up to the present time no one has found any method of avoiding the *Zermelo's axiom* {*inertial system*} other than by way of the *computability theory** {*field theory* [of General Relativity]}.

3. EINSTEIN ABOUT THE FOUNDATIONS OF THE NOTION OF SPACE. A BALANCE OF THE SUGGESTED CORRESPONDENCES

Let us notice some inaccuracies of Einstein's reflection. He does not advice the reader that never he refers to a mathematics including idealistic notions, but only to an operative, constructive mathematics. In period 4 Einstein's writing in an implicit way shifts from the distinction relational meaning / absolute meaning of space to the distinction of operative meaning /idealistic meaning of space; this distinction is revealed by his words "unlimited in extent" that may be intended in both sense. In period 5 he speaks of "geometric and kinematics viewpoints on space" when he actually has presented some descriptive aspects of space without time. Moreover he simplifies the historical process of theoretical physics leading from the notion of space to the complex subject of inertia principle. In periods 6-7 he states the indispensability for the classical physics of the absolute notion of space. In period 8 he attributes to the notion of physical field a decisive role in the history of theoretical physics, but he misses at least the space of Lagrangian coordinates and Hilbert space. Moreover, the physical field is intended by Einstein not as an idealistic notion, but in the operative way which was typical of Faraday (a step-by step interaction). Yet, in per. 9 this notion is surreptitiously changed into the idealistic notion of general relativity theory.

Table 1: The 28 substitutions performed in the above text, listed in order of appearance

Physical notions	***Mathematical notions***
[natural] *scientist*	mathematician
phenomena	mathematical objects
observation	reflection
the ascent of a mountain	a difficult calculation
hands, legs and tools	basic mathematical tools*
science of mechanics	basic mathematics
place	collection
space	set
[natural] *science*	elementary mathematics
a portion of earth's surface	an immediate gathering of familiar objects
material object or body	operative mathematical object
positional quantity	membership
material world	mathematical world
sense experience	reflection experience
geometric or kinematic viewpoints [rather, philosophical viewpoint of set] *on space*	philosophical aspects of mathematics
Descartes	Cantor

coordinate system	Coordinate of the hierarchy of the powers of infinite sets - cardinals' system
Newton	Zermelo
inertial behaviour of bodies	Zermelo's axiom of choice
Leibniz and Huygens	Gauss, Kronecker and Brouwer
causal structure of the theory	deductive structure of the theory
time	continuum hypothesis *
centuries	decades
physics	mathematics
field [= interaction step-by-step]	computable process*
Faraday and Maxwell	Skolem and Goedel*
four space-time parameters	few operations of elementary arithmetic*
covariant	Equivalent according to Turing-Church thesis*

Let us consider now the unsatisfying correspondences out of the 28 above listed correspondences. In per. 5 the correspondence does no longer concerns the two notions, but a more challenging subject, i.e. the historical roles played by notions concerning these two notions. I admit that the correspondences refer to rather than their scientific contents, the historical roles played by the notions at issue. However, they represent meaningful correspondences of the two scientific histories. For this reason I assume that they are not dubious.

Let us now examine the six dubious correspondences, each one marked by an asterisk. The first one, located in sect. 1, is of a minor importance for your analysis since concerns a mere Einstein's allusion to a generic mechanical process, which obviously is in correspondence with a mathematical calculation.

Much more relevant are the last five dubious correspondences. I remark that they are all concern the notion of field and are all located in the sect.s 8-9.

I tried to suggest a correspondence which is consistent in itself. I seem that the result is not unlikely; but it describes a constructivist Einstein's mathematical attitude that he never supported. Owing to these dubious correspondences and also his disputable appraisals in sect.s 8-9, one could dismiss these sections from our analysis, as too hazardous Einstein's reflections.

Anyway, the above 5 inaccuracies of the above parallelism are few in number with respect to the many Einstein's uncertain points of his reflection on so basic philosophical questions. Therefore the above translation substantially shows that what Einstein attributed to the physical notion of space of theoretical physics can be evenly referred to an at all different notion of a distinct scientific discipline (mathematics): set; hence, it is possible to equivalently reflect on a basic notion of theoretical physics and a basic notion of mathematics.

We have obtained a connection between the theoretical physical practice of the notion of space and the mathematical practice of the notion of set.

4. EINSTEIN'S INTRODUCTION OF TWO COUPLES OF MEANINGS OF THE NOTION OF SPACE (AND ALSO SET)

Let us re-reading Einstein's writing by focusing the attention on the content of his reflection. He distinguishes four meanings of the notion of space through two different and mutually independent distinctions of the same notion of space. One distinction is between the two meanings: "(*a*) place" as the relational Huygens's and Leibniz's relational notion of space and "(*b*) space" as Descartes' and Newton's idealized notion of an all inclusive container; i.e. idealized notion of space in Newton's mechanics [18]. Another distinction, introduced with little attention by Einstein, concerns two more meanings of the notion of space, i.e. (*c*) as it results from an operative process of accumulation of some material objects, i.e. a merely unbounded accumulation, and (*d*) as containing an actual infinity number of objects.

These distinctions may be translated *via* the above correspondences into the two distinctions between the two couples of meanings of the mathematical notion "set". The former distinction concerns the two ways of organizing a set: either as a collection of mathematical beings whose relational links are exploited by a theorist in order to perform a heuristic, or problem-based organization (PO), or as a wholeness derived from few premises (definitions or axioms). The latter distinction concerns either a set built through a process of accumulation one object after another (collection), or a set intended as containing an actual infinity number of objects, as it was conceived by Cantor. This distinction concerns the ancient one between the potential infinity and actual infinity). Today this distinction informs two kinds of set theory; either constructive set theory[19], relying on only potential infinity (it exists since some decades) and the classical set theory, freely allowing to appeal to actual infinity.

I suggest the following table 2 as resuming the above illustration.

Table 2: The correspondence between the four meanings of space and those of set

Notion	***Heuristic organization***	***Axiomatic organization***	***Potential infinity***	***Actual infinity***
Space	*a*) relational	*b*) absolute	*c*) "place", "a group of material objects", a merely unbounded accumulation	*d*) "space", an actual infinity number of accumulated objects
Set	*a'*) Collection of operative mathematical objects	*b'*) Set of naïve Cantor's set theory	*c'*) A gathering of familiar objects, a set of potential infinity number of accumu-lated mathematical objects	*d'*) A set of an actual infinity number of mathematical objects

[18] An independent, similar analysis is presented by the two papers: A. Drago (1986), "La storia del concetto di spazio quale rivelatrice delle scelte fondamentali di una teoria fisica: I. La correlazione tra lo spazio e la organizzazione della teoria", in F. Bevilacqua (ed.): *Atti VII Congr. Naz. Storia Fisica*, Padova, pp. 113-121; "La storia del concetto di spazio quale rivelatrice delle scelte fondamentali di una teoria fisica: II. A. Koyré e la metafisica della scienza moderna", *ibidem*, pp. 119-123.

[19] Markov A.A. (1962). On Constructive Mathematics. *Trudy Math. Inst. Steklov*, 67 8-14; also in *Am. Math. Soc. Translations*, (1971), 98(2), pp. 1-9. Bishop E. (1967). *Foundations of Constructive Mathematics*, New York, Mc Graw-Hill. Crosilla L. (2019), ""Set Theory: Constructive and Intuitionistic ZF", in Zalta E.N. (ed.), *The Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/set-theory-constructive/ .

In sum, he introduced two dichotomies, one about the kind of infinity of a set and another on the way of organizing a theory.

Notice that these dichotomies are entities which on one side are of a scientific nature because expressed in scientific terms; and on the other side, they are of a philosophical nature, because they cannot be decided by experimental tests. Owing to their double nature they work as interpretative categories for both philosophical and scientific subjects.

5. EINSTEIN'S PREVIOUS SUGGESTION OF THE SAME TWO DISTINCTIONS: HIS 1905 PAPER ON QUANTA

Remarkably, this philosophy of theoretical physics, based on two basic dichotomies, was manifested by Einstein not only at the end his scientific career (the above paper was written one year before his death) but also at its beginnings, in his celebrated 1905 paper on quanta[20]; there he manifested two dichotomies which are almost the same of the previous ones. In paper's "Introduction" he distinguishes in a sharp way the theoretical physics' use of the "discrete" (D) instead of the traditional use of the commonly presupposed "continuum" (C). This distinction is exactly what later he cursorily mentions in 1954 paper; the 1905 distinction confirms previous interpretation of the words "unlimited in extent" of the per. 3 of this writing as implicitly including a distinction. Remarkably, in 1905 paper Einstein explicitly calls this distinction "a dichotomy" and moreover he then solves the problem of proving the existence of discrete quanta of energy by means of constructive mathematics[21], which is based on discrete mathematics and which in general represents the alternative choice on the dichotomy on the kinds of mathematics.

Moreover, he declares that he wants to solve a problem. In the title itself of his paper Einstein claims he wants to organize his theory through a "heuristic" method, of course in opposition of the method of a deductive, axiomatic organization (I call it AO); he puts as first a problem: Do quanta exist? (I call this kind of organization PO). Since these two ways of organizing a theory, i.e. PO (heuristic) and AO are alternative one to another, also they constitute a dichotomy. In sum, he suggests two dichotomies as basic for his first theory of quanta.

It is easy to equate the latter dichotomy of the two meanings of space *c*) and *d*) with the choices of previous dichotomy D and C; the former distinction is an actualization by means of a particular basic notion of the general dichotomy between D and C, or better, between potential infinity (PI) and actual infinity (AI). The dichotomy AO/PO generalizes the distinction (*a*)/(*b*) from a basic notion to the organization of a theory; in other terms, the two philosophical meanings of the notion of space, actualizing, as Einstein says, the two kinds of organization of "material bodies", constitute a material instance of the two theoretical organizations of all the propositions of a theory, AO/PO. I feel justified by all in the above in calling these dichotomies Einstein's dichotomies, although he introduced them in different times and each time in a not complete way.

[20] Einstein A. (1905), "Ueber einen die Erzeugung der Verwandlung des Lichtes betreffenden heuristisch Gesichtpunkt", *Ann. der Physik*, 17, 132-148; reprinted in Stachel J. (ed.) (1989), *Collected Papers of Albert Einstein*, Princeton U.P., Princeton, vol. 2, 149-165.

[21] Actually, one of his calculations includes idealistic mathematical notions; yet the same translator of the paper into English language suggests a substitution of it by means of a constructive calculation (Drago A. (2013), "Einstein's 1905 "Revolutionary" Paper on Quanta as a Manifest and Detailed Example of a Principle Theory", *Advances in Historical Studies*, Vol.3, No. 3, §. 14, online: http://www.scirp.org/joumal/Paperlnibrmation.aspx?PaperID=475 19#.U7R1 ukAW2-Y ' 1)

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6. TWO DICHOTOMIES AS BASIC FOR BOTH MATHEMATICAL AND PHYSICAL PRACTICES, THE FOUR MODELS OF A SCIENTIFIC THEORY AND THE FOUR ONTOLOGIES

Being mutually independent Einstein's dichotomies may be crossed together; one obtains four models of a scientific theory. That amounts to a plurality of foundations of a scientific theory.

This pluralist foundation of physics was proved by a consistent historical reconstruction of the historical development of theoretical physics, form Galilei to Dirac (1930) (Drago 2017).

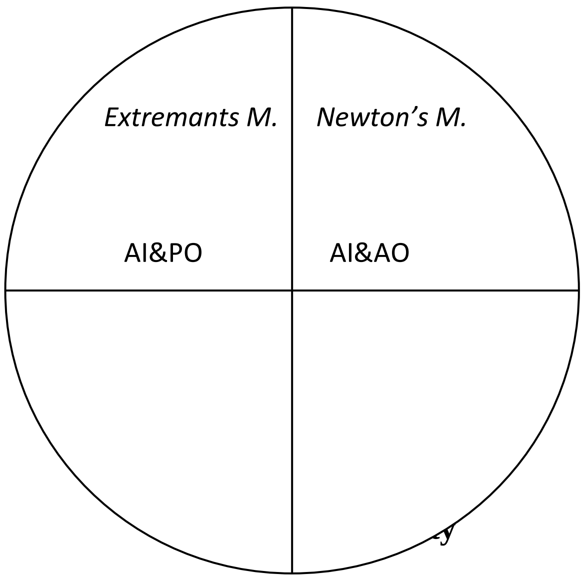
Owing to the above translation we recognize the above pluralist foundation also in mathematics.[22] Moreover, a previous paper of mine shows that the four meanings of set correspond to the four different programs in the foundations of Mathematics of the first half of the 20th century[23].

Notice that AI&AO choices of Zermelo-Fraenkel's set theory are the same of the dominant paradigm within physics, the Newtonian one. The great influence of Zermelo's set theory in mathematics since the early 1900s was similar to the great cultural influence of Newtonian paradigm in theoretical physics. It is this correspondence between the two paradigms that gives a structuralist justification of the above translation of Einstein's writing from a physical subject, space, to a mathematical subject, set.

In the following I suggest two graphical representations of the plurality of respectively the theory of mechanics[24] and set theory.

Actual Infinity

Problem-based Organization



Axiomatic Organization

Potential Infinity

[22] Hellman G. (2005), "Structuralism", in Shapiro S. (ed.), *The Oxford Handbook on Philosophy of Mathematics and Logic*, Oxford: Oxford U.P., pp. 536-562.

[23] Drago A. (2017), "A Pluralist Foundation of the Mathematics of the First Half of the Twentieth Century", *J. of Indian Council of Philosophical Research*, 34(2), pp. 343-363.

[24] For a detailed presentation of this fourfold pluralism in mechanics see Drago A. (2015), "The four prime principles of theoretical physics and their roles in the history", *Atti Fondazione Ronchi*, 70, n. 6, pp. 657-668.

Fig. 1: *Graphical representation of the pluralist foundations of theoretical physics in the case of the theory of Mechanics*

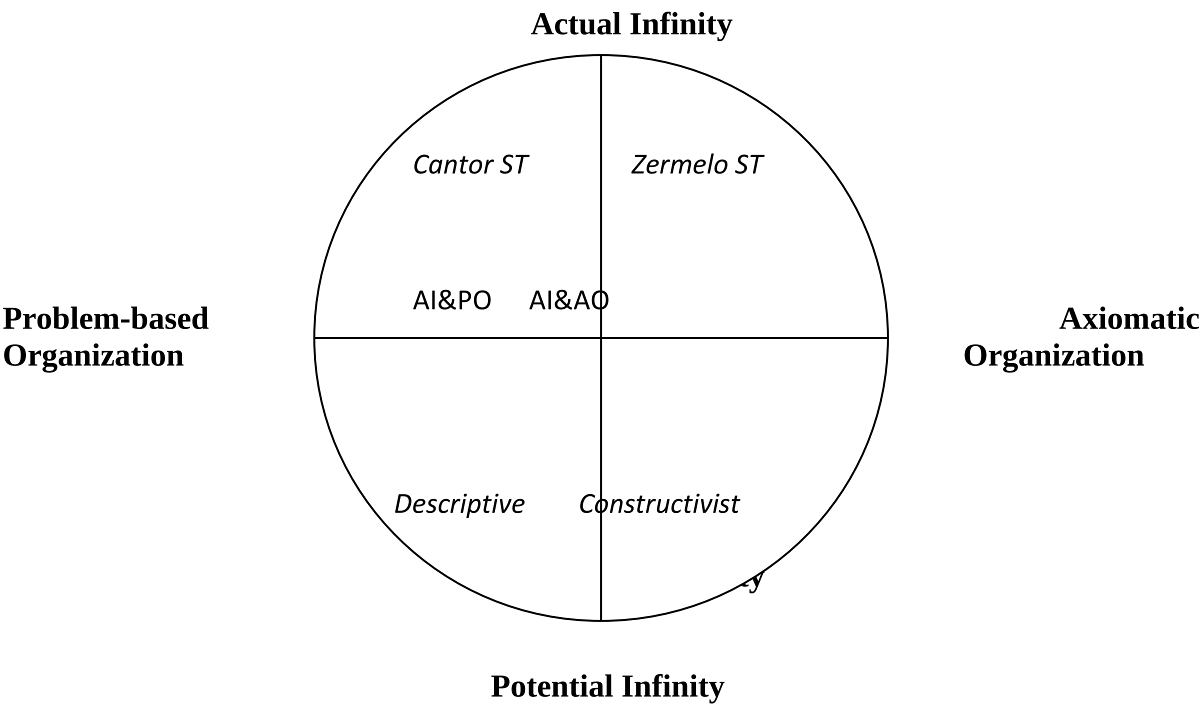


Fig. 2: *Graphical representation of the pluralist foundations of Set Theory*

This kind of foundation of mathematics suggests a new structuralism; its novelty is manifested by a comparison with all other suggestions of a structuralism[25]; the latter ones *i*) are not based on theories, *ii*) do not take into account the problem-based organization.

[25] Hellman G. (2005), "Structuralism", in Shapiro S. (ed.), *The Oxford Handbook on Philosophy of Mathematics and Logic*, Oxford: Oxford U.P., pp. 536-562.

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